
Zbl 1231.01006

Stedall, Jacqueline

From Cardano's great art to Lagrange's reflections. Filling a gap in the history of algebra. (German, English)

Heritage of European Mathematics. Zürich: European Mathematical Society (EMS). xi, 224 p. EUR 68.00 (2011). ISBN 978-3-03719-092-0/hbk
http://dx.doi.org/10.4171/092

Jacqueline Stedall opens her book with Lagrange's observation from 1771, that since Cardano there had been little progress in solving equations. Her objective is to investigate to which extent this was true and to which extent it was wrong. The pivots of the argument are four figures who, each in his own moment of history, rounded off and completed what had been done until then and at the same time opened the field for new development: Cardano himself, Viète, Newton, and Lagrange. Between these, the contributions of a large numbers of other contributors to the understanding of algebra and equation theory are discussed, some major (including Harriot, Descartes, Leibniz, Euler and Bézout) and many minor but still significant in one way or the other. Stedall challenges (p. ix) "the view that mathematics somehow progresses only by means of 'great and significant works' and 'substantial changes' " - a view that she takes to explain that not only general histories of mathematics but also works explicitly dealing with the history of algebra tend to jump directly from Descartes to Lagrange.
Part I of the book (chapters 1-3), "From Cardano to Newton: 1545-1707" is organized chronologically, and follows the emergence of various ideas and insights, first from Cardano's Ars magna over Bombelli, Stevin, Viète, Harriot and Girard until Descartes' Geometrie, and next (as the latter work had become the standard reference) in the works of Jan Hudde, Delaurens, John Collins, Gregory, Tschirnhaus, Leibniz (and several others), with culmination in Newton's Arithmetica universalis. Among these insights and ideas may be mentioned the various ways to solve cubics and quartics; the (merely partial) replacement of paradigmatic examples by literal general equations; results concerning the number and nature of roots as dependent on coefficients; Viète's introduction of proportion theory as a tool for understanding and formulating algebraic problems (which the reviewer but not always the author would distinguish from the description of the sequence of algebraic powers as a continued proportion, current among abbacus authors since the late fourteenth century and still present in Cardano and his generation) and its replacement by polynomial theory at the hands of Harriot; the transformation of polynomials and equations by way of changes of variable, and the composition of polynomials as products of linear factors; and methods for numerical approximation.
Part II (chapters 4-11) is primarily organized around a number of specific themes (each of which is then treated chronologically): determination of the number of positive, negative and complex roots of an equation from the coefficients (refinement and proofs of Descartes' and Newton's rules), involving among others Maclaurin, George Campbell, de Gua de Malves, Euler and Lagrange; roots as sums of radicals, involving John Colson, de

Moivre, Euler, Bézout and Lagrange; functions of the roots of an equation that become roots of a "resolvent equation" derived from this original equation, and from which the roots of the original equation could be hoped to be determinable (as was indeed the case for cubics and quartics), involving Euler and Bézout; elimination of variables from a system of equations, based on Newton's Arithmetica universalis and involving Euler, Cramer, Bézout and Lagrange; work on the degree of resolvent equations by Euler and Bézout, rediscovering and sharpening disappointing conclusions reached by Hudde, Gregory and Leibniz in the 17th century; and numerical solution as achieved by Newton and Lagrange. The final chapters of this part deal with "The insights of Lagrange, 1771" and the partially identical results reached by "The outsiders: Waring and Vandermonde".
A brief part III (chapter 12) describes what happened after Lagrange - primarily Ruffini's problematic proof that a general quintic cannot be solved by radicals and the continuation of this line of thought by Cauchy and Abel; Galois' and Cauchy's transformation of algebra from a theory of equations into a theory of permutation groups (tentatively intimated by Lagrange) is just hinted at in the very end.
In part I, Stedall often presents original formulations as well as translations into modern symbolism. Afterwards, the original notations are sufficiently close to what is used today to dispense with translation. This strategy, together with very sensitive discussion of the many authors involved, gives the reader an excellent impression of what really went on in the black box which, according to traditional historiography, transformed the input provided by Cardano, Bombelli, Viète and Descartes into the output generated by Lagrange.

Jens Høyrup (Roskilde)

Keywords : Cardano; Viète; Harriot; Descartes; Gregory; Tschirnhaus; Leibniz; Newton; Maclaurin; Euler; Bézout; Lagrange; Ruffini; Cauchy; equation theory
Classification :
∗01A50 Mathematics in the 18th century
01A40 Mathematics in the 15th and 16th centuries, Renaissance
01A45 Mathematics in the 17th century
12-03 Historical (field theory)
12E12 Algebraic equations